Department of science and mathematics
1st year students Hiruzen camp

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Research on Hot-Air Balloon

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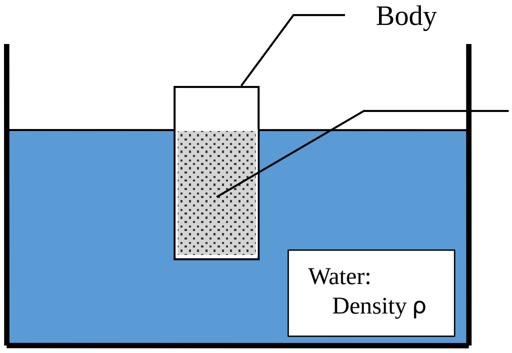
1 Principle of hot-air balloon and buoyant force

Floating force is generated when the gas inside of the balloon is heated because the gas becomes less dense.

(1) Buoyant force

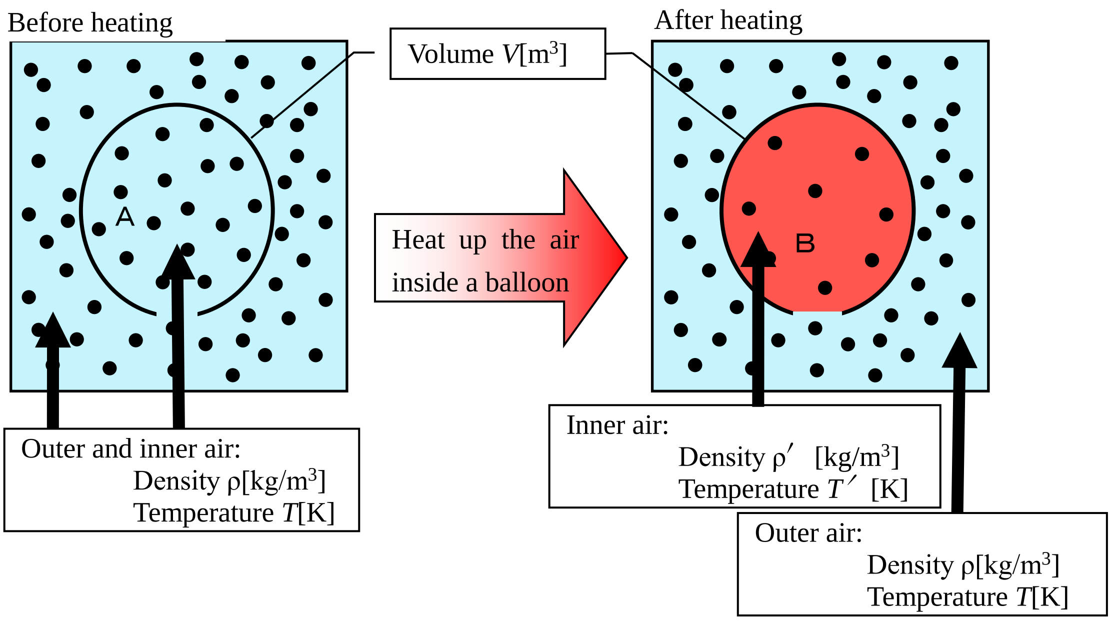
In general, buoyant force is shown as ρVg[N]. Here, ρ[kg/m^3] is the density of the fluid, V[m^3] is volume that the fluid displaced by the body, and g[m/s^2] is gravitational acceleration.

【Principle of Archimedes】
When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.



The buoyant force is same as the weight of the water displaced in this area.

At this point, ρ in ρVg is the density of water, not the density of the body.



The density of air inside a balloon decreased compared with that of outside, so the net buoyant force works on it.

(2) Calculation of net buoyant force

Here, we are going to calculate the net buoyant force of the hot-air balloon.

The forces are exerted as the figure below.

[Downward]

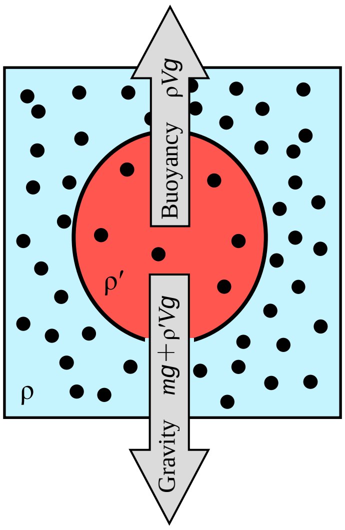
- gravity (the weight of the bag)

  m[kg] represents the mass of polyethylene(PE) bag

- gravity (the air inside)

[Upward]

- buoyant force



When buoyancy and gravity are in the equilibrium state, the equation is shown as

[Upward force] = [Downward force]

$\rho Vg = mg + \rho' Vg$

↳ become smaller when the air heated

$\rho Vg > mg + \rho' Vg$ ⇒ float

$\rho V > m + \rho' V$

$\rho - \rho' > \dfrac{m}{V}$ ・・・・・・・・①

(3) The relation between the net buoyant force and the balloon's mass and volume.

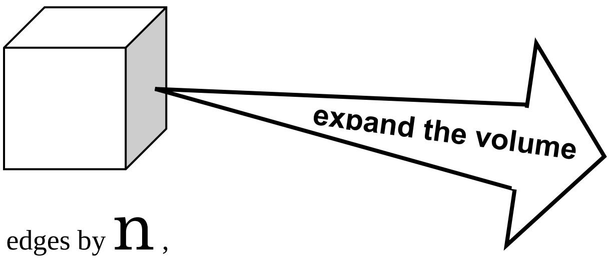
Discuss from equation①

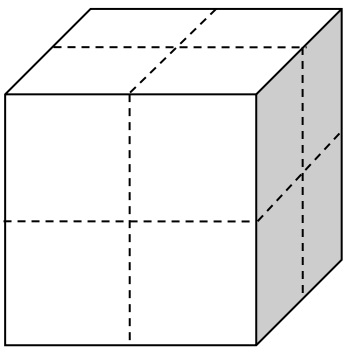
What the eq meaning: the bag will float when the difference between the densities ($\rho - \rho'$) is larger than(>) the value of the mass of bag over its volume (m/V).

The mass of the bag is proportional to its surface area.

Think in a cube, and make edges doubled. ⇒

- Surface area is (　　　)times
- Volume is (　　　) times





Multiply edges by n,

- Surface area is (　　　)times
- Volume is (　　　) times

→ As V increases, the value of $\dfrac{m}{V}$ will decrease.

【Conclusion】The balloon which has (　　　　　) volume is more buoyant.

(4) – 1 The relation between air density ρ and absolute temperature T[K]

- 22.4L in standard condition（0°C, 1atm）

Air density under standard conditions（ρ =1.293kg/m^3）※1

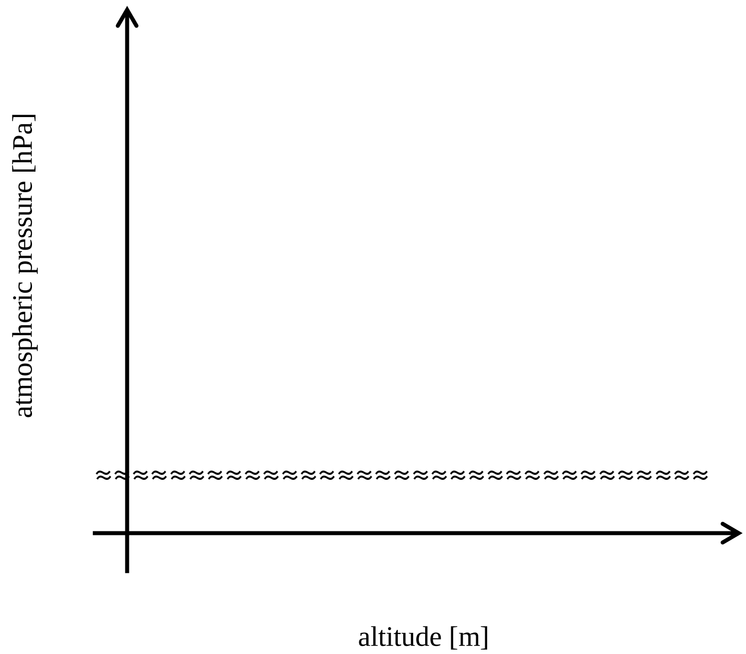
- Absolute temperature $T = t$ [°C]+273 the unit is [K] (kelvin)
- 1atm (atmospheric pressure) is same as 1013hPa.

(4) – 2 Atmospheric pressure and Altitude

The value of atmospheric pressure is equal to the weight of air per unit area acting on a point. Thus, the atmospheric pressure will decrease at high altitudes, because the volume of air above the point is less than the volume of air above a point at low altitudes.

[Activity 1] Make a graph from the numbers in the table below.

altitude [m]	pressure [hPa]
0	1013.3
200	989.5
400	966.1
600	943.2
800	920.8
1000	898.7
1200	877.2
1400	856.0
1600	835.2
1800	814.9
2000	795.0
2200	775.4
2400	756.3



[Activity 2] Compare the estimated value and actual measured value of atmospheric pressure of Hiruzen which has a height of 500m above sea level.

Estimation [ ] Actual [ ]

Derive temperature-density relational expression from Boyle-Charles' Law.

from $\underbrace{\boxed{\frac{pV}{T}}}_{\text{normal state}} = \underbrace{\boxed{\frac{p'V'}{T'}}}_{\text{specific state}}$, $\frac{p}{\frac{m}{V}T} = \frac{p'}{\frac{m}{V'}T'}$, $\frac{p}{\rho T} = \frac{p'}{\rho' T'}$, thus, $\rho' = \frac{p'}{p} \cdot \frac{T}{T'} \rho$ · · · · · ②

(5) Making a hot-air balloon

Materials: 0.015mm thick polyethylene sheet

① Make a hole with a 10 cm diameter at the center of the bottom sheet for the entrance of hot air from a dryer.

② Pair same marks, and tape all together with a Scotch tape

(6)-1 Empirical test

Equation① is conditional equation to float the hot-air balloon.

Measure the values and substitute them into equation①

m (mass of sheets) : [ ]kg

V (volume of balloon) : [ ]m^3 thus, $\frac{m}{V}$ = [ ]

T (temperature of room air) : [ ]K from equation②, $\rho =$ [ ]

T' (temperature of air inside of balloon) : [ ]K from equation②, $\rho' =$ [ ]

(6)-2 additional questions

1. Suppose that, the composition of air is $N_2 : O_2 = 4 : 1$ (N=14, O=16). Calculate and check whether the value reaches the standard density of the air (ρ=1.293kg/m^3).

2. Calculate and find the inner air temperature of the balloon to float it in current condition

Discuss about the phenomenon caused by low atmospheric pressure, and also think about its reason.

【Equipment and Materials】

・Polyethylene bag ・Scissors ・Scotch tape ・Ruler ・Electronic scale ・Electronic thermometer

・Calculator ・Gloves ・Dryer